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**Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015**  
**Engineering Mathematics – IV**

Time: 3 hrs.

Max. Marks: 100

**Note: 1. Answer any FIVE full questions, selecting at least TWO questions from each part.**  
**2. Use of statistical table is permitted.**

**PART – A**

- 1**
- a. Employ Taylor's series method to find an approximate solution to find  $y$  at  $x = 0.1$  given  $\frac{dy}{dx} = x - y^2$ ,  $y(0) = 1$  by considering upto fourth degree term. (06 Marks)
  - b. Solve the following by Euler's modified method  $\frac{dy}{dx} = \log(x + y)$ ,  $y(0) = 2$  to find  $y(0.4)$  by taking  $h = 0.2$ . (07 Marks)
  - c. Given  $\frac{dy}{dx} = x^2$  (Hy) and  $y(1) = 1$ ,  $y(1.1) = 1.233$ ,  $y(1.2) = 1.548$ ,  $y(1.3) = 1.979$ . Evaluate  $y(1.4)$  by Adams-Bashforth method. Apply corrector formula twice. (07 Marks)
- 2**
- a. Solve  $\frac{dy}{dx} = 1 + xz$  and  $\frac{dz}{dx} = -xy$  for  $x = 0.3$  by applying Runge Kutta method given  $y(0) = 0$  and  $z(0) = 1$ . Take  $h = 0.3$ . (06 Marks)
  - b. Use Picard's method to obtain the second approximation to the solution of  $\frac{d^2y}{dx^2} - x^3 \frac{dy}{dx} - x^3y = 0$  given  $y(0) = 1$ ,  $y'(0) = 0.5$ . Also find  $y(0.1)$ . (07 Marks)
  - c. Apply Milne's method to compute  $y(0.4)$  given  $y'' + xy' + y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 0$ ,  $y(0.1) = 0.995$ ,  $y'(0.1) = -0.0995$ ,  $y(0.2) = 0.9802$ ,  $y'(0.2) = -0.196$ ,  $y(0.3) = 0.956$  and  $y'(0.3) = -0.2863$ . (07 Marks)
- 3**
- a. Derive Cauchy-Riemann equation in Cartesian form. (06 Marks)
  - b. Find an analytic function  $f(z)$  whose real part is  $\frac{\sin 2x}{\cosh 2y - \cos 2x}$  and hence find its imaginary part. (07 Marks)
  - c. If  $f(z)$  is a holomorphic function of  $z$ , then show that  $\left\{ \frac{\partial}{\partial x} |f(z)| \right\}^2 + \left\{ \frac{\partial}{\partial y} |f(z)| \right\}^2 = |f'(z)|^2$ . (07 Marks)
- 4**
- a. Discuss the transformation  $w = z + \frac{1}{z}$ . (06 Marks)
  - b. Find the BLT which maps the points  $z = 1, i, -1$  to  $w = i, 0, -i$ . Find image of  $|z| < 1$ . (07 Marks)
  - c. Evaluate  $\int_C \left\{ \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} \right\} dz$  where 'C' is circle  $|z| = 3$ . (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
 2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8 = 50, will be treated as malpractice.

## PART – B

- 5 a. Express  $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$  in terms of Legendre polynomials. (06 Marks)
- b. Obtain the solution of  $x^2 y'' + xy' + (x^2 - x^2) y = 0$  in terms of  $J_n(x)$  and  $J_{-n}(x)$ . (07 Marks)
- c. Derive Rodrigue's formula  $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2 - 1)^n]$ . (07 Marks)
- 6 a. State the axioms of probability. For any two events A and B, prove that,  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ . (06 Marks)
- b. A box 'A' contains 2 white and 4 black balls. Another box 'B' contains 5 white and 7 black balls. A ball is transferred from the box A to the box B. Then a ball is drawn from the box B. Find the probability that it is white. (07 Marks)
- c. In a certain college 4% of the boys and 1% of girls are taller than 1.8m. Further more 60% of the students are girls. If a student is selected at random and is found to be taller than 1.8m, what is the probability that the student is a girl? (07 Marks)
- 7 a. The probability density of a continuous random variable is given by  $p(x) = y_0 e^{-|x|}$ ,  $-10 < x < \infty$ . Find  $y_0$ . Also find the mean. (06 Marks)
- b. Obtain the mean and variance of binomial distribution. (07 Marks)
- c. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and SD of 60 hours. Estimate the number of bulbs likely to burn for.
- More than 2150 hours.
  - Less than 1950 hours.
  - More than 1920 hours but less than 2160 hours.
- Given  $A(1.5) = 0.4332$ ,  $A(1.83) = 0.4664$ ,  $A(2) = 0.4772$ . (07 Marks)
- 8 a. In a city 'A' 20% of a random sample of 900 school boys had a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions is significant? Why? (06 Marks)
- b. A manufacturer claimed that at least 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them were faulty. Test his claim at a significance level of 1% and 5%. (07 Marks)
- c. A set of five similar coins is tossed 320 times and the result is

No. of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution  $[x_{0.05}^2 = 11.07 \text{ for } 5df]$ .

(07 Marks)

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**Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015**  
**Graph Theory and Combinatorics**

Time: 3 hrs.

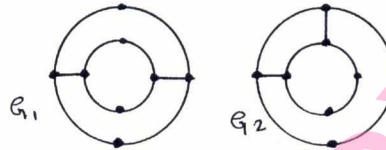
Max. Marks:100

**Note: Answer any FIVE full questions, selecting  
atleast TWO questions from each part.**

**PART – A**

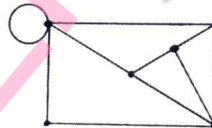
- 1 a. Define graph isomorphism and isomorphic graphs. Determine whether the following graphs are isomorphic or not: (05 Marks)

Fig.Q.1(a)



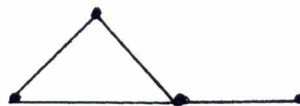
- b. Define complement of a simple graph. Let  $G$  be a simple graph of order  $n$ . If the size of  $G$  is 56 and the size of  $\bar{G}$  is 80. What is  $n$ ? (05 Marks)
- c. Let  $G = (V, E)$  be a connected undirected graph. What is the largest possible value for  $|V|$  if  $|E| = 19$  and  $\deg(v) \geq 4$  for all  $v \in V$ ? (04 Marks)
- d. Write a note on “Konigsberg bridge problem and its solution”. (06 Marks)
- 2 a. Define planar graph. Prove that the Peterson graph is nonplanar. (05 Marks)
- b. Define Hamilton cycle. How many edge disjoint Hamilton cycles exist in the complete graph with seven vertices? Also draw the graph to show these Hamilton cycles. (05 Marks)
- c. Define dual of a planar graph. Construct the dual of the planar graph given in Fig.Q.2(c). (04 Marks)

Fig.Q.2(c)



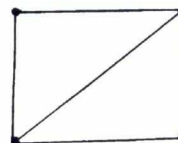
- d. Define chromatic number and chromatic polynomial. Determine the chromatic polynomial for the graph show in Fig.Q.2(d). (06 Marks)

Fig.Q.2(d)



- 3 a. A class room contains 10 micro computer that are to be connected to a wall socket that has 2 outlets. Connections are made by using extension cords that have 2 outlets each. Find the least number of cords needed to get these computer set up for use. (04 Marks)
- b. Apply merge sort to the list -1, 0, 2, -2, 3, 6, -3, 5, 1, 4. (04 Marks)
- c. Find all the spanning trees of the graph shown in Fig.Q.3(c). Also find all the non isomorphic spanning trees. (06 Marks)

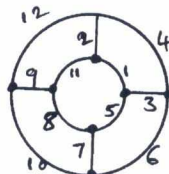
Fig.Q.3(c)



- d. Obtain an optimal prefix code for the message MISSION SUCCESSFUL. Indicate the code for the message. (06 Marks)

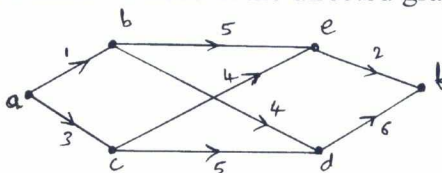
- 4 a. State Krushkal's algorithm. Using Krushkal's algorithm find a minimal spanning tree for the weighted graph shown in Fig.Q.4(a). (08 Marks)

Fig.Q.4(a)



- b. Apply Dijkstra's algorithm the diagram shown in Fig.Q.4(b) and determine the shortest distance from vertex a to each of the other vertices in the directed graph. (06 Marks)

Fig.Q.4(b)



- c. Define the following with one example for each: i) Cut set; ii) Edge connectivity; iii) Vertex connectivity. (06 Marks)

### PART - B

- 5 a. A bit is either 0 or 1. A byte is a sequence of 8 bits. Find: i) The number of bytes; ii) The number of bytes that begin with 11 and end 11; iii) the number of bytes that begin with 11 and do not end with 11 and iv) the number of bytes that begin 11 or end with 11. (06 Marks)
- b. How many arrangements of the letters in MISSISSIPPI have no consecutive S's? (05 Marks)
- c. Find the coefficient of  $x^0$  in the expansion of  $3\left(x^2 - \frac{2}{x}\right)^{15}$ . (05 Marks)
- d. In how many ways can we distribute 7 apples and 6 oranges among 4 children so that each child gets at least 1 apple? (04 Marks)
- 6 a. How many integers between 1 and 300 (inclusive) are  
i) Divisible by at least one of 5, 6, 8?  
ii) Divisible by none of 5, 6, 8? (06 Marks)
- b. Define derangement. Find the number of derangements of 1, 2, 3, 4. List all the derangements. (06 Marks)
- c. Five teachers  $T_1, T_2, T_3, T_4, T_5$  are to be made class teachers for five classes,  $C_1, C_2, C_3, C_4, C_5$  one teacher for each class.  $T_1$  and  $T_2$  do not wish to become the class teachers for  $C_1$  or  $C_2$ ,  $T_3$  and  $T_4$  for  $C_4$  or  $C_5$ , and  $T_5$  for  $C_3$  or  $C_4$  or  $C_5$ . In how many ways can the teachers be assigned the work? (08 Marks)
- 7 a. Find the generating function for the sequence 8, 26, 54, 92.... (06 Marks)
- b. Using generating function, find the number of i) non negative and ii) positive integer solutions of the equation  $x_1 + x_2 + x_3 + x_4 = 25$ . (08 Marks)
- c. Define exponential generating functions using exponential generating function find the number of ways in which 5 of the letters in the word CALCULUS be arranged. (06 Marks)
- 8 a. The number of bacteria in a culture is 1000 (approximately) and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. (05 Marks)
- b. Solve the recurrence relation  $a_{n+2} - 4a_{n+1} + 3a_n = -200, n \geq 0$  and  $a_0 = 3000, a_1 = 3300$ . (07 Marks)
- c. Find the generating function for the recurrence relation  $a_{n+1} - a_n = n^2, n \geq 0$  and  $a_0 = 1$ . (08 Marks)

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## Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015

### Design and Analysis of Algorithms

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting at least TWO questions from each part.**

#### PART – A

1.
  - a. Find gcd(31415, 14142) by applying Euclid's algorithm. Estimate how many times it is faster when compared to the algorithm based on consecutive integer checking. **(04 Marks)**
  - b. Compare the order of growth of  $\frac{1}{2}n(n-1)$  and  $n^2$ . **(04 Marks)**
  - c. Explain the mathematical analysis of fibonacci recursive algorithm. **(06 Marks)**
  - d. Write Bruteforce string matching algorithm. **(06 Marks)**
2.
  - a. Find the upper bound of recurrences given below by substitution method.
    - i)  $2T\left(\frac{n}{2}\right) + n$                       ii)  $T\left(\frac{n}{2}\right) + 1$  **(06 Marks)**
  - b. Sort the following elements using merge sort. Write the recursion tree.  
70, 20, 30, 40, 10, 50, 60 **(06 Marks)**
  - c. Write the algorithm for quick sort. Derive the worst case time efficiency of the algorithm. **(08 Marks)**
3.
  - a. Write greedy method control abstraction for subset paradigm. **(04 Marks)**
  - b. Using greedy method, trace the following graph to get shortest path from vertex 'a' to all other vertices. **(06 Marks)**

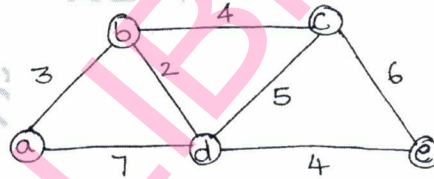


Fig. Q3 (b)

- c. What is the solution generated by the function job scheduling (JS) when  $n = 5$ ,  $[P_1, P_2, P_3, P_4, P_5] = [20, 15, 10, 5, 1]$  and  $[d_1, d_2, d_3, d_4, d_5] = [2, 2, 1, 3, 3]$  **(06 Marks)**
- d. Apply PRIMS algorithm for the following graph to find minimum spanning tree. **(04 Marks)**

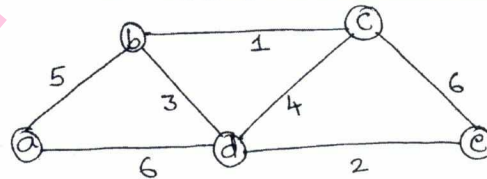


Fig. Q3 (d)

4.
  - a. Using dynamic programming, compute the shortest path from vertex 1 to all other vertices. **(10 Marks)**

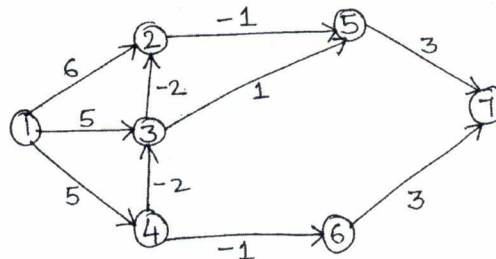


Fig. Q4 (a)

- 4 b. Solve the Knapsack instance  $n = 3$ ,  $\{W_1, W_2, W_3\} = \{1, 2, 2\}$  and  $\{P_1, P_2, P_3\} = \{18, 16, 6\}$  and  $M = 4$  by dynamic programming. (04 Marks)
- c. For the given graph, obtain optimal cost tour using dynamic programming. (06 Marks)

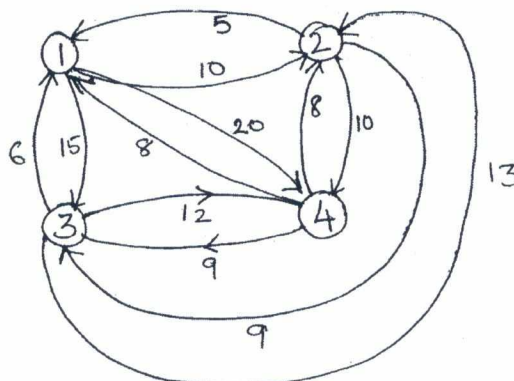


Fig. Q4 (c)

**PART - B**

- 5 a. What are the three variations of decrease and conquer technique. (03 Marks)
- b. Conduct DFS for the following graph: (05 Marks)

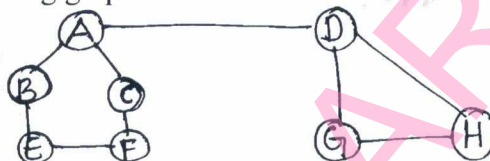


Fig. Q5 (b)

- c. Apply DFS based algorithm to solve topological sorting problem for the following graph: (06 Marks)

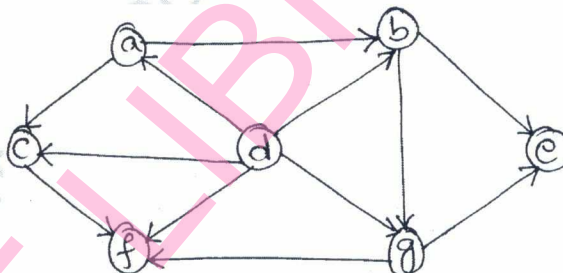


Fig. Q5 (c)

- d. Construct shift table for the pattern EARN and search for the same in text FAIL – MEANS – FIRST – ATTEMPT – IN – LEARNING using Horspool algorithm. (06 Marks)
- 6 a. Explain the four methods used to establish lower bounds of algorithm. (08 Marks)
- b. Define decision trees. Write the decision tree for the three element selection sort. (06 Marks)
- c. Define P, NP and NP complete problems. (06 Marks)
- 7 a. Explain how back tracking used for solving 4-queens problem. Write the state space tree. (06 Marks)
- b. Solve the following assignment problem using branch and bound method. (08 Marks)

	Job1	Job2	Job3	Job4
Person a	9	2	7	8
Person b	6	4	3	7
Person c	5	8	1	8
Person d	7	6	9	4

- 7 c Apply twice-around-the-tree algorithm for the travelling sales person problem for the following graph. (06 Marks)

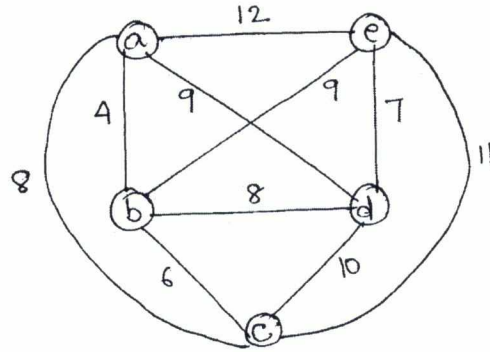


Fig. Q7 (c)

- 8 a. Explain the various models for parallel computations. (09 Marks)  
 b. Let the i/p to the prefix computation be 5, 12, 8, 6, 3, 9, 11, 12, 1, 5, 6, 7, 10, 4, 3, 5 and there are four processors and  $\oplus$  stands for addition. With diagram explain how prefix computation is done by parallel algorithm. (08 Marks)  
 c. Explain how matrix M is computed using parallel algorithm for the given graph. (03 Marks)

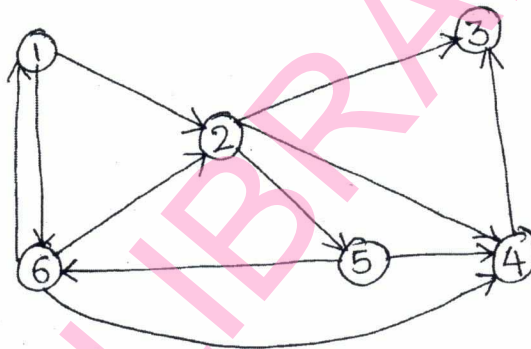


Fig. Q8 (c)

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**Fourth Semester B.E. Degree Examination, Dec.2014/Jan. 2015**  
**UNIX and Shell Programming**

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting  
atleast TWO questions from each part.**

**PART – A**

- 1
  - a. Describe briefly the UNIX architecture explaining the role played by the kernel and shell in sharing the work load. (08 Marks)
  - b. Draw the tree structure of the file system created by the following commands (assume you are in the directory/usr/office). Why is it not possible to issue the command rmdir/usr/office/right.  

```
$ mkdir left
$ mkdir middle
$ mkdir right
$ cd left
$ mkdir left middle right
$ cd ../middle
$ mkdir dir1 dir2/usr/office/right/dir3.
```

(08 Marks)
  - c. Explain the concept of absolute path name and relative pathname. (04 Marks)
  
- 2
  - a. Which command is used for listing file attributes? Explain briefly the significance of each field of the output. (06 Marks)
  - b. Assuming that a file's current permissions are r w x r -- r - x, specify the chmod expression required to change them to :
    - i) r w x r w x r - x
    - ii) r - xr - x -- x
    - iii) --- r -- r - x
    - iv) --- r w - r --,
 using both relative and absolute methods of assigning permissions. (08 Marks)
  - c. Explain the three modes of vi and explain how you can switch from one mode to another. (06 Marks)
  
- 3
  - a. Explain the three sources of standard input and standard output. (06 Marks)
  - b. Explain what these wild - card patterns match :
    - i) [A - Z]????\*
    - ii) \*[0 - 9]\*
    - iii) \*[!0 - 9]
    - iv) \*.[!s] [!h].(08 Marks)
  - c. What is a process? Mention briefly the role of fork - exec mechanism in process creation. (06 Marks)
  
- 4
  - a. What are hard-links? Explain two application areas of hard-links. What are the two main disadvantages of the hard-link? (06 Marks)
  - b. Explain these commands with examples : i) umask ii) touch . (06 Marks)
  - c. Explain the following commands :
    - i) pr ii) tail iii) sort iv) tr.(08 Marks)



## PART – B

- 5 a. Explain the grep command with options. (08 Marks)  
b. What is sed? Explain addressing in sed, with suitable examples. (08 Marks)  
c. Explain the anchoring characters. (04 Marks)
- 6 a. Explain the special parameters used by the shell. (06 Marks)  
b. What is shell script? Explain the following statements with syntax and examples :  
i) if ii) case iii)while. (10 Marks)  
c. What is the exit status of a command and where is it stored? (04 Marks)
- 7 a. Explain awk's build-in variables. (06 Marks)  
b. Write a program in awk to store the totals of the basic pay, da, hra and gross pay of the sales and marketing people. (06 Marks)  
c. Briefly describe the built-in functions in awk, with examples. (08 Marks)
- 8 a. Write a Perl script to determine whether the given year is a leap year or not. (07 Marks)  
b. Write a Perl script to convert decimal number to binary. (07 Marks)  
c. Explain variables and operators in Perl. (06 Marks)

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**Fourth Semester B.E. Degree Examination, Dec.2014/Jan. 2015**  
**Microprocessors**

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting  
atleast TWO questions from each part.**

**PART – A**

- 1 a. Write any four differences between read mode and protected mode memory system. (04 Marks)
- b. Explain real mode memory system of a personal computer system. (06 Marks)
- c. Explain briefly the programming model of 8086 through core – 2 microprocessor. (10 Marks)
- 2 a. What is paging? Explain the paging mechanism used in 80386 through core – 2 microprocessors. (08 Marks)
- b. Explain with examples, the various data related addressing modes. (08 Marks)
- c. For DS = 1200h, DI = 2024h, ARRAY = 0012h, BX = 1012 h, find the physical address for the following instructions. i) MOV AL, ARRAY [BX] ii) MOV AL, ARRAY [BX] [DI]. (04 Marks)
- 3 a. Describe the operation of the following instructions with examples :  
i) LEA ii) XLAT iii) DAA iv) IMUL. (08 Marks)
- b. Write the machine code for the following instructions :  
i) MOV BP, SP  
ii) MOV WORD PTR [BX + 1000 h], 1234h. (08 Marks)
- c. Explain the following assembler directives, with examples :  
i) EXTRN ii) PTR. (04 Marks)
- 4 a. With format explain rotate instructions. Give examples to rotate right by 2 – bit and rotate left by 4 – bits. (06 Marks)
- b. Explain the following statements to control the flow of the program, with examples :  
i) •IF – •ENDIF  
ii) •REPEAT – •UNTIL  
iii) •WHILE - •ENDW. (06 Marks)
- c. Explain the sequence of operation that takes place during the execution of CALL and RET instructions. Mention the differences between :  
i) near and far procedures  
ii) procedures and macros. (08 Marks)

**PART – B**

- 5 a. Write an 8086 ALP to find the factorial of a given number using recursive procedures. (06 Marks)
- b. Write a mixed ALP with 'C' to perform a simple calculator operations. (10 Marks)
- c. Write an 8086 ALP to convert the given binary number into its equivalent unpacked decimal and ASCII. (04 Marks)

- 6 a. Explain the following pin functions of 8086 microprocessor :  
i) READY ii) INTR iii)  $\overline{\text{TEST}}$  iv) NMI. (08 Marks)
- b. With a neat diagram, explain the minimum mode system of 8086 microprocessor. (07 Marks)
- c. Explain the timing diagram of read operation in 8086 microprocessor. (05 Marks)
- 7 a. How 8086 microprocessor selects 8 – bit or 16 – bit data from odd or even memory banks? (04 Marks)
- b. Interface  $8\text{K} \times 8$  ROM and  $4\text{K} \times 8$  RAM to 8086 microprocessor. Assume that the starting address for ROM is 40000 h and that for RAM is 44000 h. (10 Marks)
- c. Mention the differences between memory mapped I/O and Isolated I/O. (06 Marks)
- 8 a. With a neat block diagram, explain 82C55 PPI. Write the control words for  
i) PORT A input, PORT B output and PORTC output (08 Marks)  
ii) PORT A output, PORT B input, and PORTC input in simple I/O mode. (06 Marks)
- b. With a neat diagram, explain 8254 PIT. (06 Marks)
- c. Explain briefly the interrupt vector table of 8086 microprocessor. (06 Marks)

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**Fourth Semester B.E. Degree Examination, Dec.2014/Jan. 2015**  
**Computer Organization**

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting  
atleast TWO questions from each part.**

**PART – A**

- 1 a. Explain the basic operational concepts between the processor and the memory. (08 Marks)
- b. How to measure the performance of the computer? Explain. (06 Marks)
- c. Write a note on byte addressability, big-endian and little – endian assignment. (06 Marks)
- 2 a. What is an addressing mode? Explain any four types of addressing modes, with example. (08 Marks)
- b. With example, explain subroutine stack frame. (06 Marks)
- c. Explain how to encode the instructions into 32 – bit words. (06 Marks)
- 3 a. What is an interrupt? With example illustrate the concept of interrupts. (06 Marks)
- b. Explain in detail, the situations where a number of devices capable of initiating interrupts are connected to the processor? How to resolve the problems? (08 Marks)
- c. Explain the two approaches for bus arbitration. (06 Marks)
- 4 a. Describe how a read operation is performed on a PCI bus. (10 Marks)
- b. List the sequence of events that takes place when a processor sends a commands to the SCSI controller. (10 Marks)

**PART – B**

- 5 a. Discuss the internal organization of a  $2M \times 8$  asynchronous DRAM chip. (10 Marks)
- b. Describe the different mapping functions in cache. (10 Marks)
- 6 a. Write the logic diagram of 4 – bit carry look ahead adder. Explain the operation. (06 Marks)
- b. Perform multiplication for  $-13$  and  $+9$  using Booth's algorithm. (06 Marks)
- c. Write the circuit arrangement for binary division. Perform the restoring division for the given binary numbers  $1000 \div 11$ , show all the cycles. (08 Marks)
- 7 a. Explain the three – bus organization of the processor. (08 Marks)
- b. Discuss the organization of hardwired control unit. (08 Marks)
- c. Write the micro-routine for the instruction Add – (Rsrc), Rdst. (04 Marks)
- 8 a. With a neat diagram, explain the organization of a shared memory multiprocessor. (08 Marks)
- b. What is hardware multithreading? Explain the three approaches to hardware multithreading. (08 Marks)
- c. Explain : i) SISD ii) MIMD iii) SIMD and iv) SPMD. (04 Marks)

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**Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015**  
**Advanced Mathematics – II**

Time: 3 hrs.

Max. Marks:100

**Note: Answer any FIVE full questions.**

- 1 a. If  $l, m, n$  are the direction cosines of a line then prove that  $l^2 + m^2 + n^2 = 1$  (06 Marks)  
 b. Find angle between any two diagonals of a cube. (07 Marks)  
 c. Find angle between two lines whose direction cosines satisfy the equations,  $l + m + n = 0$  and  $2l + 2m - mn = 0$ . (07 Marks)
- 2 a. With the usual notations derive the equation of the plane in the form  $lx + my + nz = 0$ . (06 Marks)  
 b. Find the equation of the plane through  $(1, 2, -1)$  and perpendicular to the planes  $x + y - 2z = 5$  and  $3x - y + 4z = 12$ . (07 Marks)  
 c. Find the shortest distance between the lines,  

$$\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$$
 and  

$$\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$$
 (07 Marks)
- 3 a. Prove that  $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{c} \cdot \vec{a}) - \vec{c}(\vec{a} \cdot \vec{b})$ . (06 Marks)  
 b. Find the sine of angle between the vectors  $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$ . (07 Marks)  
 c. Show that the vectors  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + 4\hat{j} - \hat{k}$  are coplanar. (07 Marks)
- 4 a. Find the unit normal vector to the space curve  $\vec{r} = 4 \sin t \hat{i} + 4 \cos t \hat{j} + 3t \hat{k}$ . (06 Marks)  
 b. A particle moves along the curve  $\vec{r} = \cos 2t \hat{i} + \sin 2t \hat{j} + t \hat{k}$ . Find the velocity and acceleration at  $t = \frac{\pi}{8}$  along  $\sqrt{2} \hat{i} + \sqrt{2} \hat{j} + \hat{k}$ . (07 Marks)  
 c. Find angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $x = z^2 + y^2 - 3$  at  $(2, -1, 2)$  (07 Marks)
- 5 a. Find the directional derivative of  $x^2yz^3$  at  $(1, 1, 1)$  in the direction of  $\hat{i} + \hat{j} + 2\hat{k}$ . (06 Marks)  
 b. If  $\vec{F} = (x + y + 1)\hat{i} + \hat{j} - (x + y)\hat{k}$  then show that  $\vec{F} \cdot \text{curl } \vec{F} = 0$  (07 Marks)  
 c. Show that the vector  $\vec{F} = (3x^2 - 2yz)\hat{i} + (3y^2 - 2zx)\hat{j} + (3z^2 - 2xy)\hat{k}$  is irrotational. (07 Marks)

- 6 a. Prove that  $L[\sin at] = \frac{a}{s^2 + a^2}$ . (05 Marks)
- b. Find  $L[\sin t \sin 2t \sin 3t]$ . (05 Marks)
- c. Find  $L[te^{-t} \sin 2t]$ . (05 Marks)
- d. Find  $L\left[\frac{e^{at} - e^{bt}}{t}\right]$ . (05 Marks)
- 7 a. If  $L[f(t)] = \int_0^{\infty} e^{-st} f(t) dt$  then prove that  $L[f''(t)] = s^2 L[f(t)] - sf(0) - f'(0)$ . (05 Marks)
- b. Find  $L^{-1}\left[\frac{s+2}{s^2 - 4s + 13}\right]$ . (05 Marks)
- c. Find  $L^{-1}\left[\frac{s+1}{(s-2)^3}\right]$ . (05 Marks)
- d. Find  $L^{-1}\left[\log\left(\frac{s-a}{s-b}\right)\right]$ . (05 Marks)
- 8 a. Using Laplace transform solve  $y'' - 2y' + y = e^{2t}$  with  $y(0) = 0$ ,  $y'(0) = 1$ . (10 Marks)
- b. Using Laplace transform solve the simultaneous equation,  
 $\frac{dx}{dt} + y = \sin t$   
 $\frac{dy}{dt} + x = \cos t$   
 given that  $x(0)=1$ ,  $y(0) = 0$  (10 Marks)

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